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An Efficient Krylov-Subspace-Based Algorithm to Solve the Dielectric-Waveguide Problem

Kaladhar Radhakrishnan and Weng Cho Chew

***Abstract*—An efficient scheme based on the bi-Lanczos algorithm has been developed for analysis of the dielectric-waveguide problem. A two-dimensional finite-difference scheme in the frequency domain is used to discretize the waveguide cross section. The resulting sparse eigenvalue problem is solved efficiently using the bi-Lanczos algorithm. Apart from solving the modes of the dielectric waveguide, a scheme to solve for the fields in the presence of a localized source is also described. Numerical results are also included to confirm the validity of the method.**

***Index Terms*—Bi-Lanczos algorithm, finite difference, optical waveguides.**

I. INTRODUCTION

In recent years, advances in optical waveguide technology have established the need for numerical algorithms to carry out the modal analysis for dielectric waveguides. Dielectric waveguides used in integrated optics consist primarily of rectangular dielectric cores. Since waveguides of rectangular cross section have no closed-form solution, the eigenmodes of the waveguide have to be found numerically. Several numerical methods are available to solve for the modes of dielectric waveguides. The dielectric waveguides were first analyzed using the mode-matching technique [1]. Goell [2] analyzed the same problem by expanding the field using circular harmonics. More recently, with the increase in the computational power of the computers, finite-element [3], [4] and finite-difference [5]–[8] techniques were used to solve the dielectric-waveguide problem. Schweig *et al.* [5] used the $E_z - H_z$ formulation to solve the problem. However, this formulation suffered from the occurrence of spurious modes. To avoid the spurious modes, Bierwirth *et al.* [6] used the transverse-field components to formulate the problem. This results in a sparse asymmetric matrix that is free of

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spurious modes. This matrix is then rearranged into a banded matrix and solved for the eigenmodes using a standard EISPACK package. In [8], the eigenvalue problem is solved using the Chebyshev–Arnoldi algorithm.

In this paper, we use the transverse-field components to obtain the finite-difference formulation for the dielectric-waveguide problem. Instead of using the Arnoldi algorithm, which performs explicit reorthogonalization at every iteration, we use the bi-Lanczos algorithm. This reduces the computational complexity of the algorithm while retaining all the attractive features of a Krylov-subspace-based method.

II. Theory

The transverse components of the vector wave equations are used as the equations governing the electromagnetic field in an inhomogeneously filled waveguide. The space–time dependence of the electromagnetic field is assumed to be $\exp[i(k_z z - \omega t)]$. The spatial derivatives are approximated using finite differences to express the governing equation in the form of a matrix equation.

A. Finite-Difference Formulation

From the source-free Maxwell's equations, the following discretized vector wave equations can be derived:

$$\hat{\nabla} \times \mu^{-1}_{\mathbf{m}+(1/2)} \tilde{\nabla} \times \tilde{\mathbf{E}}_{\mathbf{m}} - \Omega^2 \epsilon_{\mathbf{m}} \tilde{\mathbf{E}}_{\mathbf{m}} = 0 \tag{1}$$

$$\tilde{\nabla} \times \epsilon^{-1}_{\mathbf{m}} \hat{\nabla} \times \hat{\mathbf{H}}_{\mathbf{m}+(1/2)} - \Omega^2 \mu_{\mathbf{m}+(1/2)} \hat{\mathbf{H}}_{\mathbf{m}+(1/2)} = 0. \tag{2}$$

In the above equation, $\tilde{\nabla}\times$ is the curl operator using forward difference to approximate the derivatives while $\hat{\nabla}\times$ represents the curl operator using backward difference to approximate the derivatives. The nomenclature used in this formulation is described in [9].

Matching the transverse-field components in the vector wave equations [4] yields

$$\begin{aligned} &\mu_{\mathbf{m}} \hat{\nabla}_s \times \mu^{-1}_{\mathbf{m}+(1/2)} \tilde{\nabla}_s \times \tilde{\mathbf{E}}^s_{\mathbf{m}} - \tilde{\nabla}_s \epsilon^{-1}_{\mathbf{m}} \hat{\nabla}_s \cdot \epsilon_{\mathbf{m}} \tilde{\mathbf{E}}^s_{\mathbf{m}} \\ &\quad - k^2 \tilde{\mathbf{E}}^s_{\mathbf{m}} + k_z^2 \tilde{\mathbf{E}}^s_{\mathbf{m}} \\ &= 0 \end{aligned} \tag{3}$$

$$\begin{aligned} &\epsilon_{\mathbf{m}+(1/2)} \tilde{\nabla}_s \times \epsilon^{-1}_{\mathbf{m}} \hat{\nabla}_s \times \hat{\mathbf{H}}^s_{\mathbf{m}+(1/2)} - \hat{\nabla}_s \mu^{-1}_{\mathbf{m}+(1/2)} \tilde{\nabla}_s \\ &\quad \cdot \mu_{\mathbf{m}+(1/2)} \hat{\mathbf{H}}^s_{\mathbf{m}+(1/2)} - k^2 \hat{\mathbf{H}}^s_{\mathbf{m}+(1/2)} + k_z^2 \hat{\mathbf{H}}^s_{\mathbf{m}+(1/2)} \\ &= 0. \end{aligned} \tag{4}$$

where the subscript s represents the transverse component of the field vectors or the curl operator.

Equations (3) and (4) can be expressed in the form of an eigenvalue problem as follows:

$$\overline{\mathcal{L}}_e \cdot \tilde{\mathbf{E}}_s - k_z^2 \tilde{\mathbf{E}}_s = 0 \tag{5}$$

$$\overline{\mathcal{L}}_h \cdot \hat{\mathbf{H}}_s - k_z^2 \hat{\mathbf{H}}_s = 0. \tag{6}$$

Both matrices $\overline{\mathcal{L}}_e$ and $\overline{\mathcal{L}}_h$ are asymmetric, extremely sparse, and share the same eigenvalues. Each eigenvector of these matrices corresponds to the transverse components of the electric and magnetic fields for a particular mode. Iterative solutions are desirable to solve such eigenvalue problems since they exploit the sparsity of the matrix while limiting memory requirements and computational complexity. Explicit storage of the matrix is avoided and it is accessed in the form of a matrix vector multiply.

III. Eigenvalue Problem

Krylov-subspace-based methods are desirable to solve the eigenvalue problems for sparse matrices. Arnoldi and bi-Lanczos [10] are the two Krylov-subspace-based methods available to solve the asymmetric eigenvalue problem. The Arnoldi method is widely used because of its robust nature. However, for large matrices, it can be prohibitively expensive, as the iteration vectors are reorthogonalized after every iteration. The bi-Lanczos method tends to be faster, as it does not perform explicit reorthogonalization after each iteration. However, it is not as stable as the Arnoldi method, as the iteration vectors tend to lose their orthogonality. Different reorthogonalization schemes have been proposed to overcome this problem [11]. However, despite the loss of orthogonality, it was observed that the dominant eigenpairs can be restored from the tridiagonal matrix without performing any reorthogonalization.

A. Bi-Lanczos Algorithm

The bi-Lanczos algorithm is an extension of the Lanczos algorithm for an asymmetric matrix. The original matrix $\overline{\boldsymbol{A}}$ of size $N \times N$ is reduced into an asymmetric tridiagonal matrix $\overline{\mathbf{T}}$ of the form

$$\overline{\mathbf{T}} = \begin{bmatrix} \alpha_1 & \gamma_1 & 0 & \dots & 0 \\ \beta_1 & \alpha_2 & \gamma_2 & \ddots & \vdots \\ 0 & \beta_2 & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \gamma_{M-1} \\ 0 & \dots & 0 & \beta_{M-1} & \alpha_M \end{bmatrix}$$

where M is much smaller than N. Approximations to the eigenmodes of the original matrix can be obtained from the tridiagonal matrix and iteration vectors. The algorithm generates two sets of iteration vectors $\mathbf{V}_i = [\mathbf{v}_1, \dots, \mathbf{v}_i]$ and $\mathbf{W}_i = [\mathbf{w}_1, \dots, \mathbf{w}_i]$, which satisfy the orthogonality condition $\mathbf{w}_i^T \cdot \mathbf{v}_i = \delta_{ij}$. At each iteration, the relation between the iteration vectors and matrices can be summarized as

$$\overline{\mathbf{A}} \cdot \overline{\mathbf{V}}_i = \overline{\mathbf{V}}_i \cdot \overline{\mathbf{T}}_i + \left[0, \dots, 0, \beta_i \mathbf{v}_{i+1}\right] \tag{7}$$

$$\overline{\mathbf{A}}^T \cdot \overline{\mathbf{W}}_i = \overline{\mathbf{W}}_i \cdot \overline{\mathbf{T}}_i^T + \left[0, \dots, 0, \gamma_i \mathbf{w}_{i+1}\right]. \tag{8}$$

where $\overline{\mathbf{A}} \in \mathbb{R}^{n \times n}$, $\overline{\mathbf{V}}_i, \overline{\mathbf{W}}_i \in \mathbb{R}^{n \times i}$, and $\overline{\mathbf{T}}_i \in \mathbb{R}^{i \times i}$.

B. Method I

To solve for the eigenpairs of $\overline{\mathcal{L}}_e$, we carry out the bi-Lanczos iterations to reduce it into a tridiagonal matrix $\overline{\mathbf{T}}$. The eigenvalues of the tridiagonal matrix are approximations to the eigenvalues of the matrix $\overline{\mathcal{L}}_e$. The right eigenvectors $\overline{\mathbf{X}}$ of the matrix $\overline{\mathcal{L}}_e$ are related to the right eigenvectors $\overline{\mathbf{Y}}$ of the tridiagonal matrix by the expression $\overline{\mathbf{X}} = \overline{\mathbf{V}} \cdot \overline{\mathbf{Y}}$.

The number of iterations M appears to scale as $\sqrt{N}$. Since each iteration involves two matrix-vector products, i.e., $\overline{\mathcal{L}}_e \cdot \mathbf{x}$ and $\overline{\mathcal{L}}_e^T \cdot \mathbf{x}$, the cost per iteration is $O(N)$. Thus, the cost for M iterations becomes $O(N^{1.5})$. If all the eigenvectors of the $M \times M$ tridiagonal matrix are desired, the computational cost is $O(M^3)$ or $O(N^{1.5})$. The total storage requirement is $(M + m + 4)N + 2M^2$, where m is the number of desired eigenmodes.

C. Method II

The method described in the previous section performs a full spectral decomposition of the tridiagonal matrix and, hence, solves for M eigenpairs of the matrix $\overline{\mathcal{L}}_e$. The main drawback in that method is the storage requirement, which scales as $O(N^{1.5})$, making it inefficient for solving large problems. In many applications, typically just the first few

dominant modes are desired. In this section, we present a more efficient method to solve for the dominant modes without the storage constraints of the first method.

In this technique, the iteration vectors are discarded as the bi-Lanczos algorithm is carried out and only the tridiagonal matrix is generated. The next step is to solve for just the eigenvalues of the tridiagonal matrix. Once the eigenvalues of the desired modes have been found, they are then used in conjunction with the BiCG method to solve for the eigenvectors of the desired modes. If λ_d and $\mathbf{x}_d$ are the eigenvalue and the eigenvector of the desired eigenpair, then

$$\left(\overline{\mathcal{L}}_e - \lambda_d \overline{\mathbf{I}}\right) \cdot \mathbf{x}_d = 0. \tag{9}$$

The BiCG method can be used to solve for the null space of the matrix $(\overline{\mathcal{L}}_e - \lambda_d \overline{\mathbf{I}})$. The right-hand side is set to a very small vector instead of zero for stability reasons when the BiCG iterations are carried out. The BiCG iterations have to be carried out separately for each eigenpair. The storage requirements for this algorithm is just $O(N)$ since the iteration vectors are discarded as they are generated.

IV. Localized Current Source Response

The previous sections describe methods to solve for the modes in a dielectric waveguide. An extension of these methods can be used to solve for the field in a dielectric waveguide for a given localized current source. The algorithm to solve for the fields is a generalization of the spectral Lanczos decomposition method (SLDM) for asymmetric matrices.

In the presence of a localized current source, i.e., $\tilde{\mathbf{J}}$ at $z = z'$, the transverse vector wave equation of the electric field can be written as

$$\overline{\mathcal{L}}_e \cdot \tilde{\mathbf{E}}_s + \frac{\partial^2}{\partial z^2}\tilde{\mathbf{E}}_s = \tilde{\mathbf{s}}_e \delta\left(z - z'\right). \tag{10}$$

In the above expression, the z-dependence of the fields has been suppressed and the vector $\mathbf{s}_e$ is defined as

$$\tilde{\mathbf{s}}_e = \hat{\nabla}_s (i\Omega\epsilon)^{-1} \hat{\nabla}_s \cdot \tilde{\mathbf{J}}_s - i\Omega\mu\tilde{\mathbf{J}}_s \tag{11}$$

The generalized formal solution for (10) in terms of matrix functions is

$$\tilde{\mathbf{E}}_s = \frac{1}{2i}\overline{\mathcal{L}}_e^{-1/2} \cdot e^{i\overline{\mathcal{L}}_e^{1/2}|z-z'|} \cdot \tilde{\mathbf{s}}_e \tag{12}$$

By generalizing the SLDM method [12] for asymmetric matrices, we can derive an expression for solving matrix functions as

$$f\left(\overline{\mathcal{L}}_e\right) \cdot \mathbf{v}_1 = \overline{\mathbf{V}} \cdot \overline{\mathbf{Q}} \cdot f(\overline{\mathbf{\Lambda}}) \cdot \overline{\mathbf{Q}}^{-1} \cdot \mathbf{e}_1. \tag{13}$$

where $\overline{\mathbf{Q}}$ and $\overline{\mathbf{\Lambda}}$ are the eigenpairs of the tridiagonal matrix created by the bi-Lanczos iterations. The matrix $\overline{\mathbf{V}}$ represents the iteration vectors generated during the bi-Lanczos iterations.

To evaluate (12) using (13), we first carry out the bi-Lanczos iterations and then perform a spectral decomposition of $\overline{\mathbf{T}}$. The number of iterations is observed empirically to scale as $\sqrt{N}$. This makes the overall complexity $O(N^{1.5})$. If we use (13) to compute the field in the entire xy-plane, we need to store the entire $M \times N$ matrix $\overline{\mathbf{V}}$, which contains the iteration vectors. This makes the storage requirements scale as $O(N^{1.5})$. However, in most applications, we need to solve for the field only at certain receiver locations. In that case, we need to store just the elements corresponding to those receiver locations in each iteration vector. This reduces the size of the matrix $\overline{\mathbf{V}}$ to $M \times n_R$, where n_R is the number of receiver locations. For values of n_R much less than N, the overall storage requirement scales as just $O(M^2) \sim O(N)$.

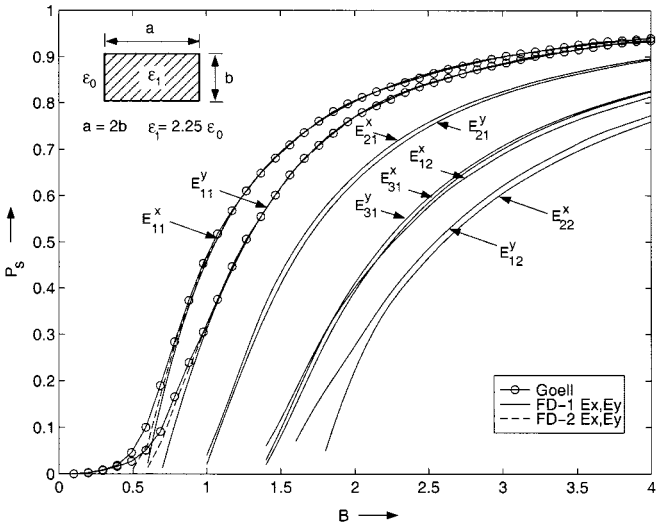

Fig. 1. Dispersion curves for the first nine propagating modes in a rectangular waveguide.

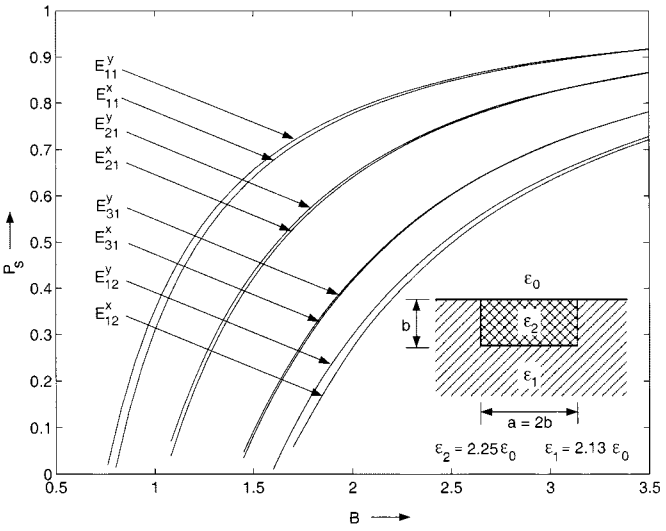

Fig. 2. Dispersion curves for the first eight propagating modes in a channel waveguide.

V. Numerical Results

In the first example, we solve for the modes in a rectangular dielectric waveguide with an aspect ratio of two and relative permittivity $\epsilon_r = 2.25$. The solution space is discretized into an uniform 120×60 grid and the problem is solved using method I, which solves for all the modes. The same problem is then discretized into a much bigger grid (480×240) and then solved for just the first two modes using method II. Fig. 1 shows the dispersion curves obtained from both methods. The normalized propagation constant P_s and the normalized frequency B are defined as

$$P_s = \frac{\left(k_z^2 - k_0^2\right)}{\left(k_1^2 - k_0^2\right)} \quad B = \frac{b}{\pi}\left[k_1^2 - k_0^2\right]^{1/2}. \tag{14}$$

Good agreement is seen between our results and those of Goell [2].

In the following example, we analyze a channel waveguide with the rectangular waveguide embedded in a substrate. The solution space is discretized into an 135×72 uniform grid. The dispersion curves for

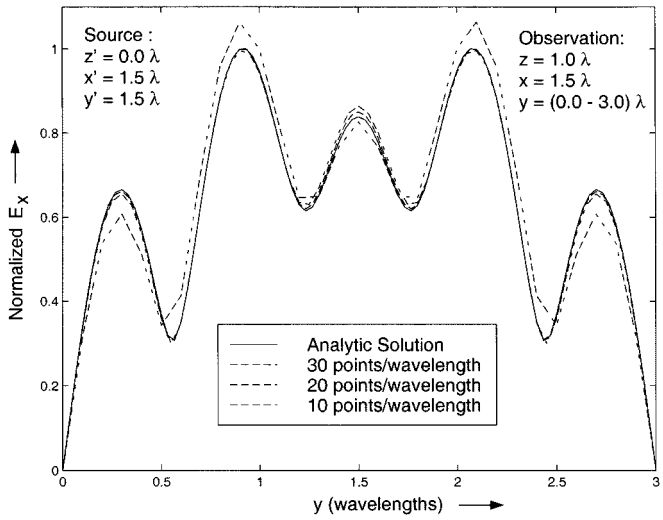

Fig. 3. Point source response for a square waveguide of size $3\lambda \times 3\lambda$ at different discretization levels.

the first few propagating modes of the channel waveguide are shown in Fig. 2.

To test the accuracy of the localized current source response, we compared our results for a homogeneously filled square waveguide with the analytic solution. A square waveguide of size $3\lambda \times 3\lambda$ was discretized at three different levels (10, 20, and 30 points/λ). A point current source was placed at the center of the waveguide and the field was computed at a distance of 1λ from the source. Fig. 3 plots the field along the y-axis center cut for the different discretization levels and compares them with the analytic solution obtained using the dyadic Green's function for a rectangular waveguide. The results are not very good when just ten points per wavelength are used and get worse as the wave is propagated down the waveguide. However, at a grid density of 30 points per wavelength, excellent agreement is seen with the analytic solution.

VI. Conclusions

We have developed an algorithm to solve the eigenvalue problem for the sparse matrix generated by the finite-difference formulation. The use of bi-Lanczos algorithm allows this method to be computationally competitive with other approximate methods, while the use of the finite-difference formulation makes this method versatile enough to handle complicated waveguide structures. We have also described a new technique that reduces the storage requirements to $O(N)$ and, thus, allows us to solve problems with several 100 000 unknowns. We have also described a scheme to solve for the localized current source response using an extension of the SLDM technique.

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A Low-Power-Consuming SOM for Wireless Communications

M. Ghanevati and A. S. Daryoush

***Abstract*—This paper presents theoretical and experimental results of a low-power-consuming hybrid push–pull self-oscillating mixer (SOM) circuit at the UHF frequency band. The frequency-stable SOM circuit is designed and fabricated using matched-pair Si bipolar junction transistors and high-Q resonators, where measured phase noise of this free-running voltage-controlled oscillator is −101.2 dBc/Hz at 100-kHz offset. A 20-dB up-conversion gain, a compression dynamic range (CDR) of 65 dB · MHz, and a spurious-free dynamic range of 50 dB · MHz$^{2/3}$ are also measured for the mixer portion of this SOM. Moreover, a down-conversion gain of $\approx$ −2 dB with a CDR of 100 dB · MHz is also measured.**

***Index Terms*—Low phase-noise oscillator, nonlinear modeling, push–pull amplifier, self-oscillating mixer, Si BJT, UHF.**

I. Introduction

Low-power-consuming front-end electronic circuits are important elements in future mobile communication systems [1]. Self-oscillating mixer (SOM) circuits that combine both local oscillators (LOs) and mixer functions have found interests in RF transceivers since they exhibit a smaller size and potentially a lower overall power consumption [2]–[7] as opposed to their discrete counter parts. A low-power-consuming SOM design topology based on the push–pull concept was first demonstrated at X-band [7] and was later modified and extended to the

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